E-x-p-a-n-d-e-d Learning Progressions Frameworks for K-12 Mathematics: A Companion Document to the *Learning Progressions Frameworks Designed for Use with The Common Core State Standards in Mathematics K-12*

*Part 2 - Grades 5-8**

This document presents expanded grade-level views of the *Learning Progressions Frameworks for grades 5-8 Mathematics* to show how smaller learning progressions can be drawn from the Learning Progressions Framework (LPF) using the Progress Indicators and highlighted links to the Common Core State Standards (CCSS) in mathematics. Unlike the original K-12 LPF document, this document is organized by smaller grade spans (5-6 and 7-8). (*Two other expanded version documents are also available: Part 1 for elementary school/ grades K-4, and Part 3 for high school/grades 9-12.) Each of these expanded versions displays all six LPF mathematics strands "unpacked" for ease of use by teachers and curriculum and assessment developers. The original document upon which they are based - *Learning Progressions Frameworks Designed for Use with the Common Core State Standards in Mathematics K-12,* including a more complete explanation of the research-based rationale and conceptual underpinnings - can be found at http://www.nciea.org/publications/Math_LPF_KH11.pdf *or at www.naacpartners.org.*
Other support materials *and related publications can be found at www.nciea.org or at www.naacpartners.org.*

Karin K. Hess, Editor
January 2011

The contents of this document were developed under a grant from the Department of Education (PR/Award #: H373X100002, Project Officer, Susan.Weigert@Ed.gov). However, the contents do not necessarily represent the policy of the Department of Education and no assumption of endorsement by the Federal government should be made.

Table of Contents

Two other LPF expanded version mathematics documents are available for elementary school/ grades K-4, and high school/grades 9-12, the original document on which they are based - *Learning Progressions Frameworks Designed for Use with the Common Core State Standards in Mathematics K-12,* and other support materials *and related publications* can be found at *www.nciea.org*.

Important CCSS note: All glossary, table, and footnote references embedded in the CCSS standards refer back to the original *Common Core State Standards for Mathematics* document which can be located at http://www.corestandards.org/the-standards/mathematics

Overview: The Learning Progressions Frameworks (LPFs) Development Process

The approach used to identify the content progressions and specific standards within the *Common Core State Standards* (CCSS) considered three important dimensions. First, national content experts and researchers in mathematics were asked to identify specific content strands that represented a way to organize essential learning for all students, K-12. Next, the committee was asked to describe the "enduring understandings" (as defined by Wiggins and McTighe, 2005) for each particular content strand, as well as articulate what the learning targets would look like if students were demonstrating achievement of the enduring understandings at the end of each grade span (K-4, 5-8, and 9-12). The grade span learning targets for each strand are stated as broader performance indicators (e.g., Use equations and expressions involving basic operations to represent a given context; Build flexibility with whole numbers and fractions to understand the nature of number and number systems). The larger grained grade span learning targets are designed to describe progressively more complex demonstrations of learning across the grade spans and use wording similar to what one might see in performance level descriptors for a given grade or grade span. (See the original K-12 LPF document for the grade span learning targets - http://www.nciea.org/publications/Math_LPF_KH11.pdf.)

In mathematics, **six major LPF strands** were established. Below is a brief description of the six strands identified by the LPF content committee. "For each content area, these essential threads [strands] interact to build greater understanding of the discipline over time. Identifying a small number of essential threads makes the learning progression manageable for the classroom teacher in terms of tracking ongoing progress in the classroom" (Hess, 2008, p.5). It is not the intent that skills/concepts from a particular strand be taught in isolation in a linear sequence, but rather be integrated among strands, such as in a problem solving situation where students are demonstrating their understanding of measurement concepts while applying their knowledge of numbers and operations and using symbolic expression. *In other words, the LPFs should be thought of as a general map for learning, not a single route to a destination.*

- **Symbolic Expression (SE)** – Symbolic Expression, presented in this document as the first strand, is a reminder NOT to teach symbolic representations before students have begun to demonstrate conceptual understanding of what the symbols or procedures actually mean (e.g., what joining together (+) and taking apart (-) sets means; understanding relative magnitude of part-whole; that "equivalence" (=) means different names for the same number). Progress indicators for the Symbolic Expression strand should be taught in conjunction with skills and concepts described in PIs from other strands and introduced with building conceptual understanding in mind.
- **The Nature of Numbers & Operations (NO)** – The skills and concepts within the Nature of Numbers and Operations strands form the foundation - and often are the prerequisite skills and concepts - for many of the other mathematics strands. Local curriculum development efforts should always consider how the skills and concepts described in the Numbers & Operations

progress indicators can be introduced, practiced, and extended with skills/concepts found in the other strands. The third N&O strand focuses on mathematical reasoning and problem solving. These progress indicators can be integrated with many CCSS standards at each grade level using problem solving contexts. While listed under the Nature of Numbers & Operations strand, the skills and concepts described in these progress indicators could apply to concepts in different mathematics strands, such as when developing proofs in Geometry.

- **Measurement (ME)** – Progress indicators are organized under two key learning targets for each grade span**.**
- **Patterns, Relations, & Functions (PFR)** – Progress indicators are organized under two key learning targets for each grade span.
- **Geometry (GM)** – Progress indicators are organized under one key learning target for each grade span.
- **Data Analysis, Probability, & Statistics (DPS)** – Progress indicators are organized under two key learning targets for each grade span. There is minimal emphasis in the CCSS on Data Analysis, Probability, & Statistics at grades K-6. Because many of the DPS mathematics skills and concepts are essential to science and social studies instruction at these grade levels, progress indicators are included in the DPS strand to guide unit development where organizing and interpreting data is important. However, you will not find many links to the CCSS mathematics standards in this strand at the lower grade levels.

Once the content committee had established the broader grade span learning targets for each strand, they were asked to identify and describe the essential skills and concepts needed to achieve the grade span expectations; use research syntheses to establish a general order of how those skills and concepts emerge for most students; and further break down the descriptors into smaller grades spans: K-2, 3-4, 5-6, 7-8, and high school. The descriptors of related skills and concepts became what we now call the **progress indicators** and the ordering/numbering used (1a, 1b, 1c, etc.) reflects the research base used to establish a general learning continuum. This means that descriptions of earlier skills build the foundation for later skills (e.g., later within a grade level, later at the next grade level/span).

The final step in the LPF development process was to identify alignment with specific CCSS mathematics content standards in order to create guidance for a cohesive curriculum experience across grades. Sometimes multiple standards from within the smaller grade spans could be linked to the same progress indicator (PI); sometimes there was only one or no standard that aligned. For example, in some strands and grade spans you will see PI descriptors that do not link (align) with an existing CCSS standard; however, the research review identified critical learning at certain stages during the learning process or skills that may be essential for conceptual understanding and for demonstrating progress. Therefore, progress indicators with no CCSS links are also included in the LPF to guide instruction, formative assessment, and progress monitoring.

Reading and Interpreting the LPF format, progress indicators, and related standards for a Grade Span – See next page. 

***NO*: Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide use of computational strategies and algorithms.**

The statement of enduring understanding across grade spans states WHY learning the skills and concepts (and standards) listed below are important and how they are generally applied.

(K-4) Elementary School Learning Targets

E.NO-2 Build an understanding of computational strategies and algorithms:

- *Fluently add, subtract, multiply, divide, and estimate;*
- *Perform and represent operations with whole numbers, fractions, and mixed numbers;*
- *Identify multiples and factors of whole numbers.*

K-4 Elementary School Grade Span Learning Targets

- By the end of grade 4, students demonstrate and apply the skills and concepts related to Numbers & Operations in a variety of situations or problem solving contexts.
- Learning targets are the more general/broad performance descriptors associated with specific skills and concepts at each grade level.

Larger grade spans are then broken into smaller grade spans

Grades K-2	Grades 3-4
Build understanding and fluency with operations...	Build understanding and fluency with operations...
E.NO.2a representing addition and subtraction in multiple ways (composing/decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping K.OA-1, 2, 3, 4; K.NBT-1 1.OA-1, 2, 5, 6; 1.NBT-4, 5, 6 2.OA-1, 4; 2.NBT-7	E.NO.2d modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers 3.OA-1, 2, 3, 4, 5 4.OA-1, 2, 3; 4.NBT- 5, 6
E.NO.2b explaining or modeling the relationship between addition and subtraction 1.OA-3, 4 2.NBT-5, 7, 9	E.NO.2e describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutativity does not apply to subtraction or division 3.OA-7, 9; 3.NBT-2 4.OA-2
E.NO.2c working flexibly with common addition and subtraction situations K. OA-2 1. OA-3, 5, 6, 8 2.OA-1, 2; 2.NBT- 2, 5, 7	E.NO.2f identifying factors and multiples of numbers 3.OA-6 4.OA-4
	E.NO.2g recognizing fractions as one number/one quantity, rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions 3.NF-1, 2, 3a, 3c

What you see articulated in this sample LPF strand:

- "E" denotes all Elementary (K-4) progress indicators.
- Most LPF progress indicators are stated in a more general way (e.g., using many related strategies; using both addition and subtraction) than a single CCSS standard; therefore progress indicators (PIs) often align with several CCSS standards at different grade levels within the grade span. This multi-standard alignment can provide insights into potential "mini progressions" for lesson design.
- Numerous CCSS standards align with the first descriptor under K-2 and can be interpreted that this progress indicator embodies many important foundational skills for all three grade levels, K, 1, and 2. Teachers at all of these grades may need to revisit lower grade level skills (and standards) for students needing reinforcement/ extra work on prerequisite skills.
- K students would spend most of their school year working on CCSS standards: K.OA-1, 2, 3, 4; and K.NBT-1 (linked to the first PI), while grades 1 and 2 would be addressing all three PIs and the associated CCSS standards in this general/a-b-c order.

Text in blue denotes links to CCSS standards:
2.OA-1,2 means grade 2, Operations & Algebraic Thinking, standards 1 and 2
(See p. 19 of CCSS for mathematics)

The highlighting in the expanded version of the LPF shows potential smaller learning progressions (LPs) and parts of the CCSS standards that link with progress indicators.

Elementary School Learning Targets

Data Analysis, Probability, and Statistics (DPS)- *DPS-1 Gather and interpret data to answer questions related to a particular/single context. Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency.*

Progress Indicators for Grades K-2	Grade K CCSS standards	Grade 1 CCSS standards	Grade 2 CCSS standards
E.DPS.1a posing questions of interest that can be answered by counting or collecting data (e.g., concrete comparisons about students, classroom materials, science topics) *with teacher guidance*	**K.CC-5** **5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.** **K.CC-6** **6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]** **([1] Include groups with up to ten objects.)**	**1-MD-1** **1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.**	**2.MD-2** **2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.** **2.MD- 5** **5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.**
E.DPS.1b identifying and sorting data/attributes; identifying rules for classifying data/attributes	**K.MD-1** **1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.** **K.MD-2** **2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*** **K.MD-3** **3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[1]** **K.G-2** **2. Correctly name shapes regardless of their orientations or overall size.** **K.G-4** **4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts & attributes**	**1.MD-1** **1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.** **1.MD-4** **4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.** **1.G-1** **1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.**	**2.G-1** **1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[1] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.** **2.MD-10** **10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[1] using information presented in a bar graph.**

Highlighting indicates links among the Progress Indicator & one or more CCSS standard or parts of the standard(s).

The highlighting in 2 colors here illustrates two possible smaller learning progressions (LPs) for instruction & assessment for this PI. This document does not show multiple LPs with different highlighting for PIs, but different LPs may exist if you look for them by matching the highlighted CCSS language with the PI description.

Expanded Learning Progressions Frameworks for K-12 Mathematics

Middle School Strands

Grades 5 - 6

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.

M.SE-1 *Represent relationships and interpret expressions and equations in terms of a given context for determining an unknown value. Represent mathematical relationships symbolically and solve for any variable (for 1st degree equations and for common formula (literal equation); Explain how to manipulate an algebraic expression to create equivalent expressions and provide step-by-step explanations and justifications.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 5 CCSS standards
M.SE.1a using symbols (=, >, <) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations **5.NBT-3b**	**5.NBT-3b** **3.** Read, write, and compare decimals to thousandths. b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.SE.1b writing, interpreting, and using expressions, equations, and inequalities (including using brackets, parentheses, or braces) **5.OA-1, 2** **6.EE-6, 8, 9**	**5.OA-1, 2** **1.** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **2.** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	**6.EE-6, 8, 9** **6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **8.** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **9.** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
M.SE.1c maintaining equality between both sides of the equation to solve equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers **6.EE- 7**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**6.EE- 7** **7.** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

M.SE.1d interpreting and using symbols to express relationships (e.g., simple formulas - volume, area; ordered pairs, ratios, percents, positive-negative numbers, exponents) **5.MD-5b** **5.G-1** **6.RP-1, 3c** **6.NS-6a, 6b** **6.EE-1, 6**	**5.MD-5b** **5.** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **5.G-1** **1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	**6.RP-1, 3c** **1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **6.NS-6a, 6b** **6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **a.** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. **b.** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **6.EE-1, 6** **1.** Write and evaluate numerical expressions involving whole-number exponents. **6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-1 *Build flexibility using rational and irrational numbers to expand understanding of number systems: Estimate, compare, and represent numbers (fractions, decimals, and percents; integers); Use exponents to express quantities and relationships; Use integers in problem solving.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.NO.1a explaining the meaning of place value (that a digit in one place represents 10 times what it represents in the place to its right) **5.NBT-1**	**5.NBT-1** **1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.NO.1b extending place value understanding to reading (e.g., naming the values with number words, rather than "point four"), writing, comparing, and rounding decimals **5.NBT-3, 4**	**5.NBT-3, 4** **3.** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.** Use place value understanding to round decimals to any place.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.NO.1c using a variety of fractional and decimal representations and locating them on a number line **5.NBT-3a** **5.NF-1**	**5.NBT-3a** **3.** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). **5.NF-1** **1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.NO.1d representing integers (positive/negative numbers) and locating them on a number line **6.NS-5, 6c, 7a**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.NS-5, 6c, 7a** **5.** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent

		points on the line and in the plane with negative number coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **7.** Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
M.NO.1e describing, representing, and comparing absolute value relationships **6.NS-7c, 7d**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.NS-7c, d** **7.** Understand ordering and absolute value of rational numbers. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
M.NO.1f recognizing equivalence of representations using fractions, decimals, and percents and using them to solve ratio problems **6.RP-1, 3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.RP-1, 3** **1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-2 *Expand use of computational strategies and algorithms to rational numbers: Perform operations fluently with rational numbers, including fractions, decimals, and percents; Identify equivalence of indicated division and fractional parts.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.NO.2a working flexibility with common addition, subtraction, multiplication, and division situations **5.NBT-5, 6** **5.NF-5a** **6.NS-2, 3, 4**	**5.NBT-5, 6** **5.** Fluently multiply multi-digit whole numbers using the standard algorithm. **6.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **5.NF-5a** **5.** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	**6.NS-2, 3, 4** **2.** Fluently divide multi-digit numbers using the standard algorithm. **3.** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. **4.** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2). Apply and extend previous understandings of numbers to the system of rational numbers.*
M.NO.2b recognizing fractions as one number (one quantity), rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions and equivalent /non-equivalent fractions **5.NF-3**	**5.NF-3** **3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.NO.2c using operations and standard algorithms with whole numbers, fractions (unlike denominators), and decimals (to hundredths) **5.NBT-5, 6, 7** **5.NF-1, 2, 4, 7** **6.NS-1, 3**	**5.NBT-5, 6, 7** **5.** Fluently multiply multi-digit whole numbers using the standard algorithm. **6.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **7.** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **5.NF-1, 2, 4, 7** **1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 +*	**6.NS-1, 3** **1.** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.* **3.** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

	5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.) **2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	
M.NO.2d contrasting situations as additive or multiplicative	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*
M.NO.2e ordering/comparing integers and representing them on the number line **6.NS-6a, 6c, 7**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.NS-6a, 6c, 7** **6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations

		on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **7.** Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-3 *Develop metacognitive skills through making conjectures and justifying mathematical solutions and arguments: Use estimation and rounding to support reasonableness of arguments/justifications; Apply multiplicative and proportional reasoning; Make, test, and justify conjectures using mathematical concepts and models.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.NO.3a using informal and rule-based arguments, evidence, and examples (e.g., estimation, rounding, arrays, visual models, diagrams) to justify mathematical solutions **5.OA-3** **5.NBT-2, 6, 7** **5.NF-2, 4a, 5, 6, 7c** **6.RP-3a, 3d** **6.NS-1, 6c, 8** **6.EE-5**	**5.OA-3** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **5.NBT-2, 6, 7** **2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **6.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **7.** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **5.NF-2, 4a, 5, 6, 7c** **2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a	**6.RP-3a, 3d** **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **6.NS-1, 6c, 8** **1.** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.* **6.** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **8.** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **6.EE-5** **5.** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

	× *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* **5.** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **6.** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* [1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.	
M.NO.3b critiquing the mathematical arguments provided by others **5.OA-3** **5.NBT-2, 6, 7** **5.NF-2, 4a, 5** **6.RP-3a, 3d** **6.NS-1, 7b, 7d, 8** **6.EE-5**	**5.OA-3** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **5.NBT-2, 6, 7** **2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **6.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value,	**6.RP-3a, 3d** **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **6.NS-1, 7b, 7d, 8** **7.** Understand ordering and absolute value of rational numbers. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a*

	the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **7.** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **5.NF-2, 4a, 5** **2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* **4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* **5.** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.	*debt greater than 30 dollars.* **6.EE-5** **5.** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME):** Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***M.ME-1*** *Extend understanding of attributes and units: Make conversions within measurement systems; Relate measurement attributes, measures, models, and formulas.*		
Progress Indicators for Grades 5-6	**Grade 5 CCSS standards**	**Grade 6 CCSS standards**
M.ME.1a identifying and describing measurable attributes (including area, surface area, volume, fractional units, absolute value with temperature), and selecting appropriate customary or metric units of measure when solving problems **5.MD-1, 3** **6.NS-7b**	**5.MD-1, 3** **1.** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **3.** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	**6.NS-7a, 7b** **7.** Understand ordering and absolute value of rational numbers. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*
M.ME.1b recognizing relationships among units and using proportional reasoning to convert measurements from one unit to another within the same system **5.MD-1** **6.RP-3d**	**5.MD-1** **1.** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**6.RP-3d** **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
M.ME.1c recognizing how the formulas for area and volume for a variety of shapes and solids are related **6.G-1, 2, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.G-1, 2, 4** **1.** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **4.** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME):** Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***M.ME-2*** *Apply appropriate techniques, strategies, and formulas to solve problems involving measurements (including derived measurements and rates).*		
Progress Indicators for Grades 5-6	**Grade 5 CCSS standards**	**Grade 6 CCSS standards**
M.ME.2a selecting and applying appropriate standard units, tools, and level of precision in real-world measurement problems (e.g., area, surface area, volume, rate) **5.MD-1, 4** **6.G-1, 2, 4**	**5.MD-1, 4** **1.** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **4**. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	**6.G-1, 2, 4** **1.** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **4.** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
M.ME.2b using a variety of strategies (decomposing complex shapes, using formulas and models) to measure area (triangles, quadrilaterals, polygons) and volume (rectangular prisms) **5.MD-4, 5** **6.G-1, 2, 4**	**5.MD-4, 5** **4.** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	**6.G-1, 2, 4** **1.** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **4.** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

M.PRF-1 *Describe and compare situations that involve change and use the information to draw conclusions: Model contextual situations using multiple representations; Calculate rates of change for real-world situations (constant).*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.PRF.1a describing how multiplication or division changes a quantity, including with fractions or decimals **5.NF-4a, 5, 7b** **6.NS-1**	**5.NF-4a, 5, 7b** **4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)* **5.** Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.* ([1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)	**6.NS-1** **1.** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? Compute fluently with multi-digit numbers and find common factors and multiples.*
M.PRF.1b distinguishing linear from nonlinear relationships as represented in graphical and tabular representations	**5.OA-3** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the*	**6.RP-3a** **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

5.OA-3 **6.RP-3a**	*terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	
M.PRF.1c comparing two rates and evaluating them for a given situation (e.g., best value) **6.RP-1, 2, 3a, 3b** **6.EE-4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.RP-1, 2, 3a, 3b** **1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **2.** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] ([1] Expectations for unit rates in this grade are limited to non-complex fractions.) **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **6.EE-4** **4.** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities.*
M.PRF.1d using symbolic equations to summarize how the quantity of something changes **6.EE- 4, 7, 9**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**6.EE- 4, 7, 9** **4.** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. Reason about and solve one-variable equations and inequalities.* **7.** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **9.** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

M.PRF-2 *Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.PRF.2a representing, analyzing, extending, and generalizing a variety of patterns using tables, graphs, words, and symbolic rules **5.OA-3** **5.NBT-2** **6.EE-6, 9**	**5.OA-3** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **5.NBT-2** **2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**6.EE-6, 9** **6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **9.** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
M.PRF.2b relating and comparing different forms of representation and identifying functions as linear or nonlinear **5.OA-3** **6.RP-1, 2, 3** **6.EE-6, 7, 9**	**5.OA-3** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	**6.RP-1, 2, 3** **1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **2.** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] ([1] Expectations for unit rates in this grade are limited to non-complex fractions.) **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and

		the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **6.EE-6, 7, 9** **6.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **7.** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. **9.** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards **Geometry (GM):** Visualizations, spatial reasoning, and properties of two- and three-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships. ***M.GM-1*** *Apply reasoning using properties of two- and three-dimensional shapes to analyze, represent, and model geometric relationships: Classify objects based on attributes and properties and solve problems using geometric relationships and properties; Decompose figures into new figures and construct figures with given conditions; Apply concepts of parallel and perpendicular.*		
Progress Indicators for Grades 5-6	**Grade 5 CCSS standards**	**Grade 6 CCSS standards**
M.GM.1a describing and classifying plane figures based on their properties **5.G-3, 4**	**5.G-3, 4** **3.** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. **4.** Classify two-dimensional figures in a hierarchy based on properties.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.GM.1b recognizing and using properties belonging to categories and subcategories of plane figures (e.g., all rectangles have four right angles, so all squares are rectangles and have four right angles) **5.G-3**	**5.G-3** **3.** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.GM.1c demonstrating the use of a coordinate system by locating/ graphing a given point or polygon using ordered pairs **5.G-1, 2** **6.G-3**	**5.G-1, 2** **1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **2.** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**6.G-3** **3.** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.GM.1d solving area, surface area, and volume problems by composing and decomposing figures **5.MD- 5c** **6.G-1, 2, 4**	**5.MD- 5c** **5.** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	**6.G-1, 2, 4** **1.** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **2.** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found

		by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **4.** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

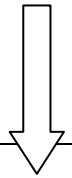
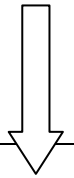
Middle School Learning Targets, Progress Indicators, & Common Core Standards

Data Analysis, Probability, and Statistics (DPS): Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.

M.DPS-1 *Design investigations and gather data to answer questions about multiple populations. Formulate questions, gather data, and build representations; Compare populations by analyzing distributions in terms of variability and measures of central tendency.*

Progress Indicators for Grades 5-6	Grade 5 CCSS standards	Grade 6 CCSS standards
M.DPS.1a formulating questions about groups larger than classroom groups and comparing different populations or samples **6.SP-1, 2**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**6.SP-1, 2** **1.** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **2.** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
M.DPS.1b distinguishing among populations, censuses, and sampling **6.SP-1, 2**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**6.SP-1, 2** **1.** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **2.** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
M.DPS.1c using representations (e.g., dot plots, scatter plots, line plots) to display data from investigations to describe the shapes of the data **5.MD-2** **5.G-2** **6.SP-2, 3, 4, 5**	**5.MD-2** **2.** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* **5.G-2** **2.** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**6.SP-2, 3, 4, 5** 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. 5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

		c. Giving quantitative measures of center (median and/or mean) and variability (inter-quartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
M.DPS.1d identifying the range, three common measures of central tendency (mean, median, and mode) and interpreting the mean as a fair share and a center of balance **6.SP-2, 3, 5c, 5d**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*	**6.SP-2, 3, 5c, 5d** 2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. 3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. 5c. Giving quantitative measures of center (median and/or mean) and variability (inter-quartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. 5d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data
M.DPS.1e making claims about populations from data distributions, supporting interpretations on the basis of mean, median, or mode, and the shape of the distribution **5.G-2** **6.SP-3, 5**	**5.G-2** **2.** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**6.SP-3, 5** **3.** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **5.** Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (inter-quartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Middle School Learning Targets, Progress Indicators, & Common Core Standards **Data Analysis, Probability, and Statistics (DPS):** Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions. ***M.DPS-2*** *Conduct probability experiments: Generate random samples to characterize variability in estimates and predictions; Analyze and build models of the association between two variables.*		
Progress Indicators for Grades 5-6	**Grade 5 CCSS standards**	**Grade 6 CCSS standards**
M.DPS.2a conducting simple probability experiments and expressing results in terms of relative frequencies or proportions as first estimate of probability	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.DPS.2b describing and representing (e.g., tree diagrams) all possible outcomes (sample space) and the theoretical probabilities of each outcome (as proportion of a specific outcome relative to all possible outcomes) in simple probability experiments		
M.DPS.2c using two-way tables to characterize distributions of two categorical variables		

Expanded Learning Progressions Frameworks for K-12 Mathematics

Elementary School Strands

Grades 7 - 8

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.

M.SE-1 *Represent relationships and interpret expressions and equations in terms of a given context for determining an unknown value. Represent mathematical relationships symbolically and solve for any variable (for 1st degree equations and for common formula (literal equation); Explain how to manipulate an algebraic expression to create equivalent expressions and provide step-by-step explanations and justifications.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.SE.1e interpreting and using symbols to express relationships or solutions (e.g., formulas; ordered pairs, ratios, exponents, squaring and cubing) **7.RP-2c, 2d** **7.G-4** **8.EE-1, 2** **8.G-9**	**7.RP-2c, 2d** **2.** Recognize and represent proportional relationships between quantities. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **7.G-4** **4.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**8.EE-1, 2** **1.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **2.** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational. **8.G-9** 9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
M.SE.1f writing and interpreting mathematical expressions, equations, and inequalities that correspond to given situations **7.EE-4a, 4b** **8.EE-8c** **8.F-3**	**7.EE-4a, 4b** **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**8.EE-8c** **8.** Analyze and solve pairs of simultaneous linear equations. c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **8.F-3** **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
M.SE.1g evaluating expressions; using expressions, linear equations,	**7.EE-4a, 4b** **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and	**8.EE-8c** **8.** Analyze and solve pairs of simultaneous linear equations. c. Solve real-world and mathematical problems leading to two linear

inequalities, and formulas to solve problems **7.EE-4a, 4b** **8.EE-8c** **8.F-3**	inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **8.F-3** **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-1 *Build flexibility using rational and irrational numbers to expand understanding of number systems: Estimate, compare, and represent numbers (fractions, decimals, and percents; integers); Use exponents to express quantities and relationships; Use integers in problem solving.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.NO.1g representing and using integers; comparing and expressing absolute value and additive inverse relationships **7.NS-1a, 1b, 1c, 2b**	**7.NS-1a, 1b, 1c, 2b** **7.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.NO.1h recognizing and modeling fractions, decimals, and percents as different representations of rational numbers **7.NS-2d** **8.NS-1**	**7.NS-2d** **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**8.NS-1** **1.** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
M.NO.1i using exponents and scientific notation to express very large or very small quantities **8.EE-1, 3, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.EE-1, 3, 4** **1.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **3.** Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the*

		population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger. **4.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
M.NO.1j making interpretations and comparisons of scientific notation produced by technology or appearing in various media **8.EE-3, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.EE- 3, 4** **3.** Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.* **4.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
M.NO.1k distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line **8.NS-1, 2**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.NS-1, 2** **1.** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **2.** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-2 *Expand use of computational strategies and algorithms to rational numbers: Perform operations fluently with rational numbers, including fractions, decimals, and percents; Identify equivalence of indicated division and fractional parts.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.NO.2f describing proportional relationships and solving related problems **(Also addressed in grade 6, 6.RP-2, 3)** **7.RP-1, 2, 3**	**(Also addressed in Grade 6)** **6.RP-2, 3** **2.** Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1] [1] Expectations for unit rates in this grade are limited to non-complex fractions. **3.** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **7.RP-1, 2, 3** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* 2. Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*

	relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	
M.NO.2g using operations with complex fractions	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.NO.2h using operations involving percents and percent increase/decrease **7.RP-3**	**7.RP-3** **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.NO.2i using operations with rational numbers; representing rational numbers and approximations of irrational numbers on a number line **7.NS-1, 2** **7.EE-3** **8.NS-1, 2**	**7.NS-1, 2** **1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational	**8.NS-1, 2** **1.** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **2.** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

	numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.EE-3** 3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

The Nature of Numbers and Operations (NO): Understandings of number - "how many" or "how much" – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

M.NO-3 *Develop metacognitive skills through making conjectures and justifying mathematical solutions and arguments: Use estimation and rounding to support reasonableness of arguments/justifications; Apply multiplicative and proportional reasoning; Make, test, and justify conjectures using mathematical concepts and models.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.NO.3c using stated assumptions, definitions, patterns, and previously established results in constructing mathematical arguments **7.RP-2a, 2d** **7.NS-1a, 2a, 3** **7.EE-3** **8.NS-2** **8.EE-6** **8.F-5** **8.SP-4**	**7.RP-2a, 2d** **2.** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **7.NS-1a, 2a, 3** **1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **3.** Solve real-world and mathematical problems involving the four operations with rational numbers.[1] [1] Computations with rational numbers extend the rules for manipulating fractions to complex fractions. **7.EE-3** **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches*	**8.NS-2** **2.** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **8.EE-6** **6.** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*. **8.F-5** **5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **8.SP-4** **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

	from each edge; this estimate can be used as a check on the exact computation.	
M.NO.3d building a logical progression of statements to explore and evaluate the truth of conjectures **7.RP-2a, 2d** **7.NS-2a** **7.EE-3** **7.G-4** **8.NS-2** **8.EE-6** **8.F-5** **8.SP-4**	**7.RP-2a, 2d** **2.** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **7.NS-2a** **2.** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. **7.EE-3** **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.G-4** **4.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**8.NS-2** **2.** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **8.EE-6** **6.** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*. **8.F-5** **5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **8.SP-4** **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME):** Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***M.ME-1*** *Extend understanding of attributes and units: Make conversions within measurement systems; Relate measurement attributes, measures, models, and formulas.*		
Progress Indicators for Grades 7-8	**Grade 7 CCSS standards**	**Grade 8 CCSS standards**
M.ME.1d applying proportional reasoning to problems with ratios of length, area, and quantities measured in like or different units **7.RP-1, 2b** **7.G-1**	**7.RP-1, 2b** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* **2.** Recognize and represent proportional relationships between quantities. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. **7.G-1** **1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.ME.1e exploring what happens to 2- and 3- dimensional measurements (such as surface area, area, and volume) when the figure is changed in some way (e.g., scale drawings) **7.G-1** **8.G-4, 9**	**7.G-1** **1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	**8.G-4, 9** **4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **9.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards **Measurement (ME):** Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events. ***M.ME-2*** *Apply appropriate techniques, strategies, and formulas to solve problems involving measurements (including derived measurements and rates).*		
Progress Indicators for Grades 7-8	**Grade 7 CCSS standards**	**Grade 8 CCSS standards**
M.ME.2c selecting and applying appropriate standard units and tools to measure to an appropriate level of precision	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.ME.2d using various strategies (decomposing complex shapes, using formulas) to measure volume (cones, cylinders, spheres) and area and circumference of circles **7.G-4** **8.G-9**	**7.G-4** **4.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	**8.G-9** **9.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
M.ME.2e solving simple problems involving scale factors, rates, and derived measures **7.G-1** **7.RP-1**	**7.G-1** **1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **7.RP-1** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.ME.2f applying the Pythagorean theorem to determine lengths/distances in real-world situations **8.G-7, 8**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.G-7, 8** **7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

M.PRF-1 *Describe and compare situations that involve change and use the information to draw conclusions: Model contextual situations using multiple representations; Calculate rates of change for real-world situations (constant).*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.PRF.1e representing and computing unit rates associated with ratios of lengths, areas, and other quantities measured in like or different units **7.RP-1, 2, 3** **8.EE-5**	**7.RP-1, 2, 3** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* **2.** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	**8.EE-5** **5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
M.PRF.1f identifying essential quantitative relationships in a situation and using symbolic expressions to represent it and draw reasonable conclusions from it **7.RP-1, 2, 3**	**7.RP-1, 2, 3** **1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.* **2.** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate	**8.EE-5, 7** **5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **7.** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different

7.EE-2, 3, 4 **8.EE-5, 7** **8.F-1, 2, 3, 4, 5**	plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. **7.EE-2, 3, 4** **2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **3.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the	numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.F-1, 2, 3, 4, 5** **1.** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] ([1]Function notation is not required in Grade 8.) **2.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **4.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

	problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	
M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations **7.RP- 2, 3** **7.EE-2, 4** **8.EE-5, 7** **8.F-2, 3, 4, 5**	**7.RP- 2, 3** **2.** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate. **3.** Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. **7.EE-2, 4** **2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the*	**8.EE-5, 7** **5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **7.** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. **8.F- 2, 3, 4, 5** **2.** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **4.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **5.** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

	solutions.	
M.PRF.1h representing and describing how rates of change can be computed from differences in magnitudes and vice versa	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

M.PRF-2 *Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.PRF.2c relating and comparing different forms of representation and identifying functions as linear or nonlinear **8.F-3**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	**8.F-3** **3.** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
M.PRF.2d solving linear equations and formulating and explaining reasoning about expressions and equations **7.EE-2, 4a** **8.EE-6, 7**	**7.EE-2** **2.** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."* **7.EE-4a** **4.** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	**8.EE-6** **6.** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **8.EE-7** **7.** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
M.PRF.2e using functions to describe quantitative relationships **8.EE-5, 7** **8.F-1, 3, 4, 5**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	**8.EE-5** **5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. **8.EE-7** **7.** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting

		like terms. **8.F-1, 3, 4, 5** 1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1] ([1]Function notation is not required in Grade 8.) 3. Interpret the equation *y = mx + b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* 4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Middle School (Grades 5-8) Learning Targets, Progress Indicators, & Common Core Standards

Geometry (GM): Visualizations, spatial reasoning, and properties of two- and three-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships.

M.GM-1 *Apply reasoning using properties of two- and three-dimensional shapes to analyze, represent, and model geometric relationships: Classify objects based on attributes and properties and solve problems using geometric relationships and properties; Decompose figures into new figures and construct figures with given conditions; Apply concepts of parallel and perpendicular.*

Progress Indicators for Grades 7-8	Grade 7 CCSS standards	Grade 8 CCSS standards
M.GM.1e constructing or drawing geometric shapes from given conditions (e.g., draw triangles given three angle or side measures; change scale) **7.G-1, 2**	**7.G-1, 2** **1.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **2.** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*
M.GM.1f recognizing and demonstrating rotations, reflections, and translations using multiple contexts (e.g., using coordinates, models, drawings, technology) **8.G-1, 2, 3, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.G-1, 2, 3, 4** **1.** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. **2.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **3.** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
M.GM.1g demonstrating congruence and similarity using a variety of two dimensional figures **8.G-2, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, these skills/concepts represent the "hypothesized" learning continuum.*	**8.G-2, 4** **2.** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **4.** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity

		between them.
M.GM.1h solving real-world area, surface area, and volume problems using different strategies (formulas and decomposing figures) **7.G-4, 6** **8.G-9**	**7.G-4, 6** **4.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **6.** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	**8.G-9** **9.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
M.GM.1i exploring and explaining angle relationships (e.g., pairs of parallel lines cut by a transversal, including perpendicular lines) **8.G-5**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.G-5** **5.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*.
M.GM.1j applying the Pythagorean Theorem **8.G-7, 8**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.G-7, 8** **7.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Middle School Learning Targets, Progress Indicators, & Common Core Standards **Data Analysis, Probability, and Statistics (DPS):** Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions. ***M.DPS-1*** *Design investigations and gather data to answer questions about multiple populations. Formulate questions, gather data, and build representations; Compare populations by analyzing distributions in terms of variability and measures of central tendency.*		
Progress Indicators for Grades 7-8	**Grade 7 CCSS standards**	**Grade 8 CCSS standards**
M.DPS.1f formulating questions about groups larger than classroom groups, comparing different populations or samples, and involving two variables **7.SP-1**	**7.SP-1** **1.** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.DPS.1g displaying and interpreting univariate data using dot plots, histograms, and circle graphs **7.SP-2, 4**	**7.SP-2, 4** **2.** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.DPS.1h displaying data in scatter plots and investigating the association between the variables **8.SP-1, 2**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concept as part of the "hypothesized" learning continuum.*	**8.SP-1, 2** **1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **2.** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.DPS.1i using box plots, interquartile range, mean absolute deviation, range, and the concept of outliers to characterize the distribution (variability) of univariate data **7.SP-2, 3, 4**	**7.SP-2, 3, 4** **2.** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

	survey data. Gauge how far off the estimate or prediction might be. **3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	
M.DPS.1j comparing two unequal distributions of data using number of data points, measures of central tendency, shape, and variability (numerical data), and two-way tables (categorical variables) **7.SP- 3, 4** **8.SP-1, 3, 4**	**7.SP- 3, 4** **3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	**8.SP-1, 2, 3, 4** **1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
M.DPS.1k supporting claims about the results of investigations (e.g., coordinating among the measures of central tendency and variability) **7.SP- 3, 4** **8.SP-1, 3, 4**	**7.SP- 3, 4** **3.** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **4.** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	**8.SP-1, 3, 4** **1.** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for

		rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Middle School Learning Targets, Progress Indicators, & Common Core Standards **Data Analysis, Probability, and Statistics (DPS):** Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions. ***M.DPS-2*** *Conduct probability experiments: Generate random samples to characterize variability in estimates and predictions; Analyze and build models of the association between two variables.*		
Progress Indicators for Grades 7-8	**Grade 7 CCSS standards**	**Grade 8 CCSS standards**
M.DPS.2d identifying sample spaces for multi-stage probability experiments (independent events) and determining the theoretical probabilities of specific event combinations **7.SP-5, 6, 7**	**7.SP-5, 6, 7** **5.** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. **6.** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **7.** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*

M.DPS.2e designing and conducting multi-stage (compound) probability experiments (independent events) and comparing the results with theoretical probabilities **7.SP- 8**	**7.SP- 8** **8.** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.DPS.2f distinguishing between association of two variables and cause and effect relationship between two variables	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level; however, instruction should include these skills/concepts as part of the "hypothesized" learning continuum.*
M.DPS.2g using simple lines to model association between two numerical variables in a bivariate data set **8.SP-2, 3, 4**	*No specific Common Core Standards have been linked to this Progress Indicator at this grade level.*	**8.SP-2, 3, 4** **2.** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **3.** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **4.** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

References

Council of Chief State School Officers & National Governors' Association. (June, 2010). *Common core state standards for mathematics.* Common core state standards initiative. Retrieved [on line] June 2010: http://www.corestandards.org/the-standards/mathematics

Hess, Karin K., (Ed.) December 2010. *Learning progressions frameworks designed for use with the common core state standards in mathematics K-12.* National Alternate Assessment Center at the University of Kentucky and the National Center for the Improvement of Educational Assessment, Dover, N.H. (updated- v.2) available [online] http://www.nciea.org/publications/Math_LPF_KH11.pdf.

Hess, K. (2008). "Developing and using learning progressions as a schema for measuring progress." [online] available: http://www.nciea.org/publications/CCSSO2_KH08.pdf.

Wiggins, G. & McTighe, J. (2005). *Understanding by design.* Alexandria, VA: Association for Supervision and Curriculum Development.

Selected Instructional Resources that Support a Learning Progressions Framework

Ainsworth, L., & Viegut, D. (2006). *Common formative assessment: How to connect standards-based instruction and assessment*. Thousand Oaks, CA: Corwin Press.

Bamberger, H. J., Oberdorf, C., & Schultz-Ferrel, K. (2010). *Math misconceptions: From misunderstanding to deep understanding.* Portsmouth, N. H.: Heinemann.

Department of Education and Training, Western Australia. Beverly, MA: STEPS Professional Development:

First Steps in Mathematics: Chance and Data (2007)

First Steps in Mathematics: Space (2007)

First Steps in Mathematics: Measurement, Volume 1 (2005) *& Volume 2* (2007)

First Steps in Mathematics: Number, Volumes 1 & 2 (2007)

Fisher, D. & Frey, N. (2007). *Checking for understanding: Formative assessment techniques for your classroom*. Alexandria, VA: Association for Supervision and Curriculum Development.

Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement*. Portsmouth, N. H.: Heinemann.

McTighe, J. & Wiggins, G. (2004). *Understanding by design: Professional development workbook.* Alexandria, VA: Association for Supervision and Curriculum Development.

National Council of Teachers of Mathematics. *Mathematics assessment: A practical handbook*. Reston, VA: National Council of Teachers of Mathematics:
Grades K-2 (2003) Glanfield, Bush, & Stenmark (Eds.)
Grades 3-5 (2001) Stenmark & Bush (Eds.)
Grades 6-8 (2000) Bush & Leinwand (Eds.)
Grades 9-12 (1999) Bush & Greer (Eds.)

New Zealand Department of Education (2007). *The number framework*. Retrieved [on line] October, 2008: http://www.nzmaths.co.nz/numeracy/2007numPDFs/NumBk1.pdf

Petit, M., Laird, R., & Marsden, E. (2010). *A focus on fractions: Bringing research to the classroom*. NY, NY: Routledge Press.

Rose, C., Minton, L., Arline, C. (2007). *Uncovering student thinking in mathematics*. Thousand Oaks, CA: Corwin Press.

Shepard, L. (2005). "Linking formative assessment to scaffolding." *Educational Leadership*, Association for Supervision and Curriculum Development, *63*(3), 66-70.

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide* (NCEE #2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from whatworks.ed.gov/ publications/practiceguides. http://ies.ed.gov/ncee/wwc/pdf/practiceguides/fractions_pg_093010.pdf

State of Victoria, Department of Education and Early Childhood Development. Victoria, Australia [online] available:
Mathematics Learning Progression:
http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm

Treacy, K. & Cairnduff (2009). *Revealing what students think: Diagnostic tasks for fractional numbers*. Beverly, MA: STEPS Professional Development.